Name: Key

- This packet is due at **12:00 pm (noon) PDT on Saturday, May 8**.
- Neatly fill out this packet either directly or on your own paper. Then upload your answers to Canvas. Please clearly number each problem and keep them in order.
- For each problem in this packet, **organize and show all of your work and justification** for your answer.
- Make sure that it is clear what your final answer is. This may mean boxing/circling it or it may mean doing neither.
- If you are required to draw a graph of a function, each axis must be labeled and given a scale.
- I will not be grading every individual problem, but answers will be available on Canvas after the due date.
- I am happy to help you however I can. Here are some options:
  - Come to my office hours (see syllabus).
  - Email me directly at gknapp4@uoregon.edu

1. Use any method to determine if the given series is convergent or divergent. Clearly state which tests were used, including making sure to check off any necessary hypotheses.

(a) $\sum_{n=1}^{\infty} \frac{2^n}{n^4}$

$\lim_{n\to\infty} \frac{2^n}{n^4} \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{2^n \ln(2)}{4n^3} = \lim_{n\to\infty} \frac{2^n \ln(2)^2}{12n^2}$

$= \lim_{n\to\infty} \frac{2^n \ln(2)^3}{24n} = \lim_{n\to\infty} \frac{2^n \cdot \ln(2)^4}{24}$

$= \infty$

By the divergence test, $\sum_{n=1}^{\infty} \frac{2^n}{n^4}$ diverges

(b) $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right) = \sum_{n=1}^{\infty} \ln(n+1) - \ln(n)$

$S_1 = \ln(2) - \ln(1)$

$S_2 = \ln(2) - \ln(1) + \ln(3) - \ln(2)$

$S_3 = \ln(2) - \ln(1) + \ln(3) - \ln(2) + \ln(4) - \ln(3)$

$\vdots$

$S_k = \ln(2) - \ln(1) + \ln(3) - \ln(2) + \cdots + \ln(k+1) - \ln(k)$

$= \ln(k+1) - \ln(1) = \ln(k+1)$

$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right) = \lim_{k\to\infty} S_k = \lim_{k\to\infty} \ln(k+1) = \infty$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+4}$

$0 \le \frac{1}{n^2+5n+4} \le \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-test ($p=2$).

Hence, by the Direct Comparison Test, $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+4}$ also converges.

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$

$0 \le \left| \frac{(-1)^n \cdot n!}{n^n} \right| = \frac{n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1}{n \cdot n \cdot n \cdots n \cdot n}$

$= \underbrace{\frac{n}{n}}_{\le 1} \cdot \underbrace{\frac{n-1}{n}}_{\le 1} \cdot \underbrace{\frac{n-2}{n}}_{\le 1} \cdots \underbrace{\frac{3}{n}}_{\le 1} \frac{2}{n} \cdot \frac{1}{n} \le \frac{2}{n^2}$

$\sum_{n=1}^{\infty} \frac{2}{n^2} = 2 \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-test ($p=2$).

By the Direct Comparison Test, $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges.

By the Absolute Convergence Test, $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n!}{n^n}$ converges.

3. Put one checkmark in each row according to how you feel about each vocab word. This is merely graded for completion and I will not judge you in the slightest for checking "no idea." Just make sure to learn the vocab before the exam!

Vocab Term	**Can Define**	**Have Seen**	**No Idea**
The divergence test			
The integral test			
p-series			
Integral test remainder estimate			
Direct comparison test			
Limit comparison test			
Alternating series			
Alternating series test			
Absolute convergence			
Conditional convergence			
Absolute convergence test			
Ratio test			
Root test			